Chapter V

Indiscernibles In Stable Theories

Sets of indiscernibles play a number of important roles in model theory. They are used to realize types (as in the proof of the existence of saturated models); they are also used to 'blow up' models (without realizing new types). Moreover the cardinalities of maximal sets of indiscernibles can be used as invariants in classifying models. We begin this chapter by expounding the basic properties of indiscernibles in a stable theory and explaining the distinction between sequences of indiscernibles and independent sets. Section 2 begins the rather lengthy process of using sets of indiscernibles as bases for models of stable theories (cf. the introduction to Section 2). In Section 3 we apply the notion of indiscernibility to show the equivalence between the notion of forking as introduced here and the original version of Shelah [Shelah 1978].

1. Sets Of Indiscernibles

If X is a set of algebraically independent elements in an algebraically closed field then every permutation of X is in fact an elementary map. This indiscernibility of the elements of X is closely related to their independence. We explain this connection in Lemma 1.8 and Theorem 1.23. In this section we study in detail indiscernible elements in a model of a stable theory. We define such notions as indiscernible sequences (of sequences) and indiscernible sets (of sequences). That is, we deal with families $E = \{\overline{e}_i : i \in I\}$ where each $\overline{e}_i$ is a finite sequence. Very little intuition is lost by thinking of each $\overline{e}_i$ as a single individual but the added generality is necessary.

1.1 Definition. The ordered set $X = \{\overline{x}_i : i \in l\}$ is a *sequence of (order) indiscernibles* if every order preserving map f mapping into a finite subset of the linear order l induces a partial elementary monomorphism of X by taking $\overline{x}_i$ to $\overline{x}_{f(i)}$.

The index set will be well-ordered unless we explicitly assert otherwise. We distinguish now between a sequence and a set of indiscernibles. While