SOLVABILITY OF SOME ASYMPTOTICALLY HOMOGENEOUS ELLIPTIC PROBLEMS

E.N. Dancer

In this talk, we discuss some problems on the existence and uniqueness of solutions of some nonlinear boundary value problems. Most of the time we will discuss existence and finally, at the end, we will discuss a non-uniqueness result.

Assume that Ω is a bounded domain in R^n, L is a self-adjoint linear operator on $L^2(\Omega)$ with compact resolvent and that $g : R \to R$ is continuous such that $y^{-1}g(y) \to \mu(\nu)$ as $y \to \infty\ (-\infty)$. We now want to discuss whether the equation

$$(1) \qquad Lu = g(u) - f$$

has a solution u in $L^2(\Omega)$ for every $f \in L^2(\Omega)$. For *simplicity*, we also assume that L is bounded below. As an example of the type of problem to which we wish to apply our results, we could take L to be $-\Delta$ with Dirichlet boundary conditions.

These type of problems have been studied in some detail in recent years. We will discuss a few of the results known and, in more detail, some of the open problems. A more complete bibliography could be obtained from the references in [3] - [6].

Let $y^+ = \sup\{y,0\}$ and $y^- = y - y^+$. Now it is easy to show that $\|u\|^{-1}(g(u)-\mu u^+-\nu u^-) \to 0$ in $L^2(\Omega)$ as $\|u\| \to \infty$. Then one might expect that there is a close relationship between the solvability of (1) for every $f \in L^2(\Omega)$ and the solvability of