APPENDIX I

DISCRETE SPECTRAL VALUES

This appendix is a supplement to Section 7. We give necessary and sufficient conditions for an isolated spectral value of $T \in BL(X)$ to be a *pole* of the resolvent operator $R(z)$. These conditions do not involve the knowledge of $R(z)$ for z near λ. They, in turn, give conditions for λ to be in the discrete spectrum of T. We show that a spectral value λ of T belongs to the discrete spectrum of T if and only if some commuting compact perturbation of T dislodges λ from the spectrum.

Let A be a linear operator on X. Consider the ascending chain of subspaces of X :

$$\{0\} \subset Z(A) \subset Z(A^2) \subset \ldots$$

and also the descending chain

$$X \supset R(A) \supset R(A^2) \supset \ldots .$$

As we have seen in Remark 7.2, if equality holds at any of the inclusions in the above two chains, then it persists at all later inclusions. This property allows us to define the following concepts. As usual, $A^0 = I$, the identity operator.

The <u>ascent</u> of A is

$$\alpha(A) = \begin{cases} 0 , & \text{if } Z(A) = \{0\} , \\ p , & \text{if } Z(A^{p-1}) \neq Z(A^p) = Z(A^{p+1}) , \ 1 \leq p < \infty , \\ \infty , & \text{otherwise.} \end{cases}$$