16. METHODS FOR INTEGRAL OPERATORS

In this section we describe some methods of approximating an integral operator of the following kind. Let X denote either $L^2([a,b])$ or $C([a,b])$, and accordingly, let $\tilde{X}$ denote either $L^2([a,b]\times[a,b])$ or $C([a,b]\times[a,b])$; we shall denote by $\| \ \|$ the L^2-norm $\| \ \|_2$ in the first case and the supremum norm $\| \ \|_\infty$ in the second. Let $k \in \tilde{X}$, and consider the <u>Fredholm integral operator</u> $T : X \to X$ <u>with kernel</u> k given by

$$(16.1) \qquad Tx(s) = \int_a^b k(s,t)x(t)dt \ , \ x \in X \ , \ s \in [a,b] \ .$$

It is well known that T is a compact operator and

$$(16.2) \qquad \|T\|_2 \le \|k\|_2 \ , \quad \|T\|_\infty \le (b-a)\|k\|_\infty \ .$$

(cf. [L], 17.5(d).) We shall also compare the methods introduced in this section with those related to projections, as described in Section 15.

Degenerate kernel method

A kernel $k \in \tilde{X}$ is said to be <u>degenerate</u> if

$$(16.3) \qquad k(s,t) = \sum_{i=1}^{m} x_i(s)y_i(t) \ , \ s,t \in [a,b] \ ,$$

where x_i and y_i belong to X, $i = 1,\ldots,m$. Notice that an integral operator with a degenerate kernel is a bounded operator of finite rank. For the kernel given by (16.3), we have for $x \in X$ and $s \in [a,b]$,