1.4. Norm-dissipative Operators.

The Hille-Yosida theorem establishes that norm-dissipativity of a generator H, i.e., the condition

$$\|(I+\alpha H)a\| \geq \|a\| , \qquad a \in D(H) ,$$

for small $\alpha > 0$, is an infinitesimal reflection of contractivity of the associated semigroup. Next we discuss a reformulation of dissipativity which corresponds to a more geometric interpretation of contractivity. This reformulation is the Banach space analogue of the condition

$$\mathrm{Re}(a, Ha) \geq 0 , \qquad a \in D(H) ,$$

which characterizes dissipative operators H on Hilbert space.

The semigroup S is contractive if, and only if, it maps the unit sphere, $\{a ; \|a\| = 1\}$, into the unit ball, $\mathcal{B}_1 = \{a ; \|a\| \leq 1\}$. Thus the change $S_t a - a$ of an element a must be toward the interior of the ball of radius $\|a\|$. To describe this last geometric idea in a quantitative manner it is necessary to introduce the notion of a tangent functional.

An element $f_a \in \mathcal{B}^*$ is defined to be a norm-tangent functional at a if

$$\|b\| \geq \|a\| + \mathrm{Re}\big(f_a, b-a\big)$$

for all $b \in \mathcal{B}$. Geometrically each such functional describes a hyperplane tangent to the graph of $b \in \mathcal{B} \longmapsto \|b\| \geq 0$ at the point a. The functional f_a divides the space into two sets