SQUARE FUNCTIONS IN BANACH SPACES

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1. INTRODUCTION

Suppose that $(T_t : t \in \mathbb{R}^+)$ is a bounded semigroup of operators on the Banach space X, of type C_0, with infinitesimal generator A. In the classical case, where (T_t) is the Poisson semigroup acting on $L^p(\mathbb{T})$ or on $L^p(\mathbb{R})$, the "g-function", developed by A. Zygmund and his school, is one of the important tools of Fourier analysis. In a more general setting, the functions $g_n(f)$

$$g_n(f)(x) = \left(\int_{\mathbb{R}^+} dt/t \, \left|t^n \, \partial^n/\partial t^n \, T_t f(x)\right|^2\right)^{\frac{1}{2}}$$

were considered by E.M. Stein [1], and used to shed light on heat diffusion semigroups, again on L^p- spaces. In particular, g-functions are often used to prove pointwise convergence results and multiplier theorems; see Stein's paper [3] for a survey of their role. Roughly speaking, the finiteness of $\|g_n(f)\|$ measures degrees of "orthogonality" of the functions $t \mapsto t^n \, \partial^n/\partial t^n \, T_t f$, for different t.

In this paper, we shall present a personal approach to g-functions, and connect them to multiplier theorems which develop Stein's work [2]. We describe the multiplier results briefly before returning to the g-functions.

For φ in $(0,\pi)$, we let Γ_φ be the following open cone:

$$\Gamma_\varphi = \{z \in \mathbb{C} : |\arg(z)| < \varphi\} .$$

We say that $H^\infty(\Gamma_\varphi)$ acts on A if there is an extension of the