INTEGRATION FOR THE SPECTRAL THEORY

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Let E be a complex Banach space. Let $B(E)$ be the algebra of all bounded linear operators on E. Then $B(E)$ is a Banach algebra with respect to the operator (uniform) norm defined by $\|T\| = \sup\{|Tx| : |x| \le 1, x \in E\}$, for every $T \in B(E)$. By I is denoted the identity operator.

A spectral measure is a multiplicative and σ-additive (in the strong operator topology) map $P : \mathcal{Q} \to B(E)$, whose domain, $\mathcal{Q}$, is a σ-algebra of sets in a space Ω, such that $P(\Omega) = I$. An operator $T \in B(E)$ is said to be of scalar type if there exists a spectral measure P and a P-integrable function f such that

(1) $$T = \int_\Omega f dP.$$

This notion, due to N. Dunford, extends to arbitrary Banach space the idea of an operator with diagonalizable matrix on a finite-dimensional space. It proved to be very fruitful as shows the exposition in the monograph [3]. Many powerful techniques in which scalar operators play a role are based on the requirements that $\mathcal{Q}$ be a σ-algebra and that P be σ-additive. But precisely these requirements are reponsible for excluding many operators of prime interest from the class of scalar-type operators. Suggestions for extending this class lead to new interesting theories.

So, C. Foias introduced the notion of a generalized scalar operator, replacing the algebra of all bounded measurable functions by some other, possibly poorer algebras of functions and the integration map by certain