INTEGRATION OF LIE ALGEBRAS

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1. Introduction

The principal question we wish to address can be informally phrased as follows:

When is a Lie algebra of closed operators on a Banach space the differential of a continuous representation of the corresponding Lie group?

An answer, expressed equally informally, can be given as follows:

Whenever an associated heat equation has a unique solution satisfying certain smoothness conditions.

The answer immediately raises a second question:

What are the minimal smoothness requirements?

The best response currently known to this latter problem is as follows:

For general group representations C_4 -conditions are sufficient but in special cases less is required, e.g. for unitary representations on Hilbert space C_8 -conditions suffice.

In order to pose these questions more precisely and to explain the answers more accurately we first introduce a number of formal definitions. Subsequently we outline the general strategies usually adopted to tackle such integrability problems. Finally we describe the various special techniques developed to solve the problems and survey various recent results in this area.