A TRANSMUTATION PROPERTY OF THE GENERALIZED ABEL TRANSFORM ASSOCIATED WITH ROOT SYSTEM A_2

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Let Σ be a root system of type A_2 in a real two dimensional Euclidean space $\mathfrak{a}$. Let W denote the Weyl group of Σ and $\mathcal{D}_W(\mathfrak{a})$ the space of W-invariant C^∞-functions on $\mathfrak{a}$ with compact support. Choose a positive Weyl chamber $\mathfrak{a}^+$. For $f \in \mathcal{D}_W(\mathfrak{a})$ and a complex parameter m with positive real part we define (as in [2]) an integral transform on $\mathfrak{a}^+$ which coincides, for certain values of the parameter m, with the Abel transform on some symmetric spaces of the noncompact type. An important property of the Abel transform is that it intertwines the radial part of the Laplace-Beltrami operator on these symmetric spaces with the ordinary Laplacian on $\mathfrak{a}$. In this note we state the result that the generalized Abel transform as introduced in [2] also satisfies this transmutation property. Detailed proofs will appear elsewhere.

In $\mathbb{R}^3$ we have the standard basis $\{e_1, e_2, e_3\}$ and inner product $\langle\cdot,\cdot\rangle$ for which this basis is orthonormal. Let $\mathfrak{a}$ denote the hyperplane in $\mathbb{R}^3$ orthogonal to the vector $e_1+e_2+e_3$. The inner product on $\mathbb{R}^3$ induces an inner product on $\mathfrak{a}$ which we shall also denote by $\langle\cdot,\cdot\rangle$. We identify the dual of $\mathbb{R}^3$ with $\mathbb{R}^3$ and the dual $\mathfrak{a}^*$ with $\mathfrak{a}$ by means of this inner product.

The root system of type A_2 can be identified with the set $\Sigma = \{\pm(e_1-e_2), \pm(e_1-e_3), \pm(e_2-e_3)\}$ in $\mathfrak{a}$. For Σ we take as basis $\Delta = \{e_1-e_2, e_2-e_3\}$ and we denote by Σ^+ the set of positive roots with respect to Δ. The positive Weyl chamber will be denoted by $\mathfrak{a}^+$. Let W denote the Weyl group of Σ. For $m \in \mathbb{C}$ we define $L(m)$, the so-called radial part of the Laplace-Beltrami operator associated with A_2, by

$$L(m) = \sum_{i=1}^{3} \frac{\partial^2}{\partial x_i^2} + m \sum_{1\le i<j\le 3} \coth(x_i - x_j) \left(\frac{\partial}{\partial x_i} - \frac{\partial}{\partial x_j} \right) . \tag{1}$$

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