1. PRELIMINARIES, NOTATION, CONVENTIONS

Even though the notation and conventions adopted here are fairly standard, slight variations that occur in the literature can cause inconvenience to the reader. So, the problem of making the whole text sufficiently self-contained is solved by placing this chapter at the beginning. None-the-less the chapter can be used as an appendix, that is, the reader may refer to it only as the need arises. To facilitate such usage, frequent references to this one are made in the subsequent chapters.

A. The need to treat real and complex vector spaces separately will only seldom arise. Therefore, the real or complex numbers will be referred to simply as numbers or scalars.

To maintain the perspicuity of the notation pertaining to vector valued functions and integrals, the multiplication by scalars of elements of a vector space will be written commutatively. That is to say, if E is a vector space, we shall write interchangeably $cx = xc$, for any scalar c and a vector $x \in E$.

By a seminorm on a vector space E is meant a function $q : E \to [0,\infty)$ such that $q(x+y) \le q(x) + q(y)$, for every $x \in E$ and $y \in E$, and $q(cx) = |c|\, q(x)$, for every $x \in E$ and a number c. So, a seminorm has all the properties of a norm with the only exception that its value may be equal to zero on a non-zero element of E.

The study of spaces of individual integrable functions, rather than those of the equivalence classes of such functions, makes it convenient to consider general seminormed and not just normed spaces. To be sure, a seminormed space is a vector space together with a specified seminorm on it. A majority of concepts referring to normed spaces are with obvious modifications applicable to seminormed spaces. The occasional difficulties are caused mainly by the non-uniqueness of limits and similar objects.

So, let E be a seminormed space with the seminorm q.

A set $S \subset E$ is called bounded if $\{q(x) : x \in S\}$ is a bounded set of numbers.