THE REDUCTION PROBLEM FOR EINSTEIN'S EQUATIONS

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I. MOTIVATION

It has long been known that the vacuum (or electro-vacuum) Einstein equations for spacetimes admitting a single (spacelike) Killing field can be *locally* reexpressed as a $2 + 1$ dimensional system of Einstein-harmonic-map equations [1, 2] . In this formulation the unknown fields are (in the vacuum case) a Lorentzian metric on the (locally defined) space of orbits of the Killing field and the norm and twist potential of this Killing field which define a harmonic map from the space of orbits to hyperbolic two-space. A natural problem is to study this formulation *globally* on suitably chosen manifolds and to apply standard methods of elliptic analysis to reduce the system to its "minimal form". In physics terminology one would like to solve an elliptic set of equations for all the dependent or "non-propagating" variables and be left with an unconstrained hyperbolic system for only the independent "physical degrees of freedom" of the gravitational field.

One mathematical motivation for this reduction program is that it aims to take maximal advantage of the elliptic aspects of Einstein's equations, for which the needed global analysis techniques are already at hand, and to reduce the complementary (and presumably more difficult) hyperbolic aspects of the system as much as possible. The ultimate aim of this project is to study the long time existence properties of Einstein's equations and the hope is that the reduction program should help to make such an analysis more tractable.