FUNCTION THEORY ON BANACH ALGEBRAS

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First let me recall some notions and results in function theory on the complex plane $\mathbb{C}$. Then by adopting suitable means they are extended to Banach algebras.

Let D be a region (an open connected set) in $\mathbb{C}$ and let H(D) be the class of all functions holomorphic in D. In general, in this work the study of univalent functions is confined to the class of functions $S = \{f \in H(E) : f(0) = 0$ and f is univalent in $E\}$ where $E = \{z \in \mathbb{C} : |z| < 1\}$ is the open unit disc in $\mathbb{C}$.

A domain D in $\mathbb{C}$ is said to be *convex* if the line joining any two points in D lies in D . A function $f \in S$ is said to be *convex* in E if f(E) is a convex set. Let K denote the collection of all convex functions in E . The analytic criteria for $f \in K$ is $\mathcal{Re}\left\{1 + z\dfrac{f''(z)}{f'(z)}\right\} > 0$ in E .

A domain D in $\mathbb{C}$ is said to be *starshaped with respect to a point* $O \in D$ if the line joining any point $a \in D$ to O lies completely in D . It is obvious that any convex domain is starshaped with respect to each of its points. A function $f \in S$ is said to be *starlike* in E if f(E) is a starshaped domain with respect to the origin. Let S^* denote the collection of all starlike functions in E . Clearly we have $K \subseteq S^*$. The analytic criteria for $f \in S^*$ is $\mathcal{Re}\left\{\dfrac{zf'(z)}{f(z)}\right\} > 0$ in E . Thus we have Alexander's theorem, namely: $f \in K$ if and only if $zf'(z) \in S^*$. Also $f \in S^*$ can be equivalently put as $(1-t)f(E) \subseteq f(E)$, for all $t \in I = [0,1]$. For details of the study of geometric function theory on the complex plane, the readers are referred to [1].

Recently another new class S_c^* of functions that are starlike with respect to conjugate points has been introduced by Thomas and El Rabha [5]. A function $f \in S$