INTERTWINING WITH ISOMETRIES

K.B. Laursen

This lecture contains work done jointly with P. Vrbová [6] and will develop some variations on a theme which goes back to Johnson and Sinclair [2].

The general question under scrutiny is that of continuity of intertwiners: if S, T are given linear operators on the vector spaces X, Y, respectively, then we consider the space

$$\mathcal{I}(S,T) := \{\theta : X \to Y \mid \theta \text{ linear}, \ C(S,T)^n\theta = 0 \ , \text{ some } n \in \mathbb{N}\} \ ,$$

where $C(S,T)^n$ is the nth composition of the map

$$C(S,T) \, : \, \theta \to S\theta - \theta T \ ,$$

and try to decide when $\mathcal{I}(S,T)$ consists of continuous maps (provided X, Y are Banach spaces and S, T are continuous).

The interest in the space $\mathcal{I}(S,T)$ stems from the fact that it contains many significant classes of maps:

If A, B are Banach algebras and $\theta : A \to B$ is an algebra homomorphism then $\theta \in \mathcal{I}(\theta(a), a)$ for any $a \in A$ in the sense that

$$\theta(a)\theta(x) - \theta(ax) = 0$$

for all $x \in A$.

Another class of examples emerges if X is a Banach algebra and Y is a commutative Banach X-module; if $D : X \to Y$ is a module derivation then $C(a,a)^2 D = 0$, as an easy calculation will show.

To state the main results we will need a few facts about the algebraic spectral subspaces $E_S(A)$ of a linear operator S on a vector space Y: given a subset $A \subseteq \mathbf{C}$,