BANACH ALGEBRAS OCCURRING IN SPECTRAL THEORY

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1. THE STONE THEOREM

The algebras referred to in the title will be specified at the end of the following section. This section is devoted to a few fairly simple observations about the Fourier L^p-multipliers. I hope that in the light of these observations the notions introduced in the following section will seem natural. At any rate, it will be apparent that the realm of mathematical objects to which these notions pertain is sufficiently rich to warrant our attention.

The algebra of all bounded linear operators on a complex Banach space, E, is denoted by $\mathrm{BL}(E)$. The identity operator is denoted by I. The norm on E is denoted as modulus and the operator (uniform) norm of an element, T, of $\mathrm{BL}(E)$ by $\|T\| = \sup\{|Tx| : |x| \leq 1, x \in E\}$.

To avoid complicated notation and circumlocution, we shall identify subsets of a given basic space with their characteristic functions.

Let λ be Lebesgue measure on $\mathbb{R}$. Let $1 < p < \infty$. Let $\mathcal{M}^p$ be the family of all (individual) functions on $\mathbb{R}$ which determine Fourier multiplier operators on the space $E = L^p(\mathbb{R})$. That is, $f \in \mathcal{M}^p$ if and only if there exists an operator $T_f \in \mathrm{BL}(E)$ such that $(T_f\varphi)\hat{} = f\hat{\varphi}$, for every $\varphi \in L^p \cap L^2(\mathbb{R})$. Here, of course, $\hat{\varphi}$ denotes the Fourier-Plancherel transform of an element, φ, of $L^2(\mathbb{R})$ and $f\hat{\varphi}$ is the point-wise product of f and $\hat{\varphi}$.

Let $\mathcal{R}^p$ be the family of all sets $X \subset \mathbb{R}$ such that $X \in \mathcal{M}^p$. Let $P^p(X) = T_X$, for every $X \in \mathcal{R}^p$.

PROPOSITION 1. *Let f be an absolutely continuous function on $\mathbb{R}$, or else, let $w > 0$ and let f be a w-periodic function on $\mathbb{R}$ which is absolutely continuous in an interval $[s,t)$ with $t - s = w$.*

Then there exist numbers c_j and Borel sets X_j, $j = 1,2,\ldots,$ such that