THE RAIKOV CONVOLUTION MEASURE ALGEBRA

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1. INTRODUCTION

What follows is propaganda for the study of a particular commutative Banach algebra, A – one not inappropriate to a conference which emphasises automatic continuity. In fact A is that subalgebra of the measure algebra, $M(\mathbb{T})$, of all regular bounded Borel measures on the circle under the total variation norm and convolution multiplication, which is characterised by the automatic continuity of measurable characters:

$$A = \{\mu \epsilon M(\mathbb{T}) : \text{ If } \nu << \mu \text{ and } \chi \in (\mathbb{T}_d)\hat{} \text{ is } \nu\text{-measurable, then } \chi \text{ is continuous}\}$$

Here $\mathbb{T}_d$ denotes the circle group with the discrete topology and $(\mathbb{T}_d)\hat{}$ its (compact) dual group.

The challenge of A is that its Gelfand structure exhibits the delicate interplay of harmonic analysis and Banach algebra theory which one finds in the full measure algebra $M(\mathbb{T})$, while the cruder pathology which arises from thin sets is necessarily absent. Indeed A admits an alternative characterization as the collection, B, of all **basic measures** defined by

$$B = \{\mu \epsilon M(\mathbb{T}) : E \text{ Borel}, \ |\mu|(E) > 0 \Rightarrow gp(E) = \mathbb{T}\}$$

Here $gp(E)$ denotes the intersection of all subgroups of $\mathbb{T}$ which contain E. By way of an exercise let us note that, for a symmetric Borel set E, some n-fold sum, $(n)E$, has positive Haar measure if and only if $gp(E) = \mathbb{T}$.