THE WEDDERBURN DECOMPOSITION FOR QUOTIENT ALGEBRAS ARISING FROM SETS OF NON-SYNTHESIS

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1. INTRODUCTION

Let B be a complex commutative unital Banach algebra and let R be the radical for B. We say B has a *Wedderburn splitting* if there exists a subalgebra C of B such that $B = C \oplus R$. If a closed subalgebra C can be found, we say B has a *strong splitting*. The question of the existence of Wedderburn splittings has been investigated in several papers. See for example [3] and [8]. There are algebras B for which no splitting exists [3, Theorem 5.2] and also algebras having an algebraic splitting but no strong splitting [2, Theorem 6.1].

Let G be a non-discrete locally compact Abelian group. Our purpose in this note is to explore the question of a Wedderburn splitting for the non-semisimple quotient algebras of $A(G)$ which arise from compact sets of non-synthesis in G. Let E be such a set of non-synthesis and $J(E)$ be the minimal ideal whose hull is E. In 1961, Katznelson and Rudin [9] moved that $B = A(G)/\overline{J}(E)$ never has an algebraic Wedderburn splitting $B = C \oplus R$ (where $R = \operatorname{rad}(B)$ in the case that G is totally disconnected and B is generated by its idempotents. In 1987, Bachelis and Saeki proved without any additional hypotheses, that $A(G)/\overline{J}(E)$ never has a strong splitting. We shall prove here that if $A(G)/\overline{J}(E)$ has an algebraic splitting, then it has a strong one, and, hence, none at all. Actually, the proof shows that $A(G)/H$ never has a splitting, whenever H is a closed ideal satisfying $\overline{J}(E) \subseteq H \subseteq K(E)$, $H \neq K(E)$.

2. PRELIMINARIES

Let A be a complex commutative semi simple Banach algebra with unit 1 and structure space Φ_A. We suppose in the present discussion that A is a Silov algebra. This