OPERATORS OF TYPE ω WITHOUT A BOUNDED H∞ FUNCTIONAL CALCULUS

Alan McIntosh [1] *and Atsushi Yagi*

1. Introduction

At the previous C.M.A. Miniconference which was held at Macquarie University, one of us considered operators T of type ω acting in a Hilbert space $\mathcal{H}$, and listed several conditions equivalent to such an operator T having a bounded H_∞ functional calculus [McI]. We shall list these again shortly. It is sometimes asked whether every operator of type ω satisfies one or other of these conditions, so we would like to take this opportunity to show that they do not. On other occasions we have considered operators T of type ω with respect to a double sector, and discussed the conditions under which such a T has a bounded H_∞ functional calculus, which implies in particular that T has bounded spectral projections associated with each sector. An example of an operator with no such bounded projections will also be presented.

In the next section we shall recall some results from [McI] and other papers. We shall then define and study some operators which will be used in the following two sections to construct the counter-examples. These examples are really modifications of those presented in earlier papers [Mc,1,2,3,4], and their existence comes as no surprise to those who are familiar with this material.

In the final section we shall show that every operator T of type ω does have a bounded H_∞ functional calculus if it is considered as acting in a Hilbert space $\mathcal{H}_T$ in which the norm of $\mathcal{H}$ is replaced by a square function norm.

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