MEAN CURVATURE EVOLUTION OF SPACELIKE HYPERSURFACES

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1. Spacelike hypersurfaces of prescribed mean curvature

Spacelike hypersurfaces of prescribed mean curvature have played an important role in the study of the structure of Lorentzian manifolds. Examples include the singularity theorems of Hawking and Penrose [HE], the analysis of the Cauchy problem for Einstein's equations based on $3+1$ foliations ([CBY], [LA]) and the first proof of the positive mass theorem due to Schoen and Yau ([SY]).

General existence and regularity results for prescribed mean curvature hypersurfaces were obtained by Gerhardt ([CG]) and Bartnik ([B1]), boundary values problems were treated in [BS] and in [B2]. For an excellent survey of the area we refer to the article by Bartnik ([B3]).

In [EH1] and [E1,2], spacelike hypersurfaces of prescribed mean curvature were constructed as stationary limits of mean curvature flow, which had previously been very successfully employed in Riemannian manifolds (see [H1] for a survey). In this talk, we will review some of the central features of this nonlinear evolution process in the special case of spacelike hypersurfaces in Minkowski space as most of the essential analytical difficulties already arise in this simplest situation.

Minkowski space $\mathbf{R}^{n,1}$ is $\mathbf{R}^{n+1}$ endowed with the metric $\langle \cdot, \cdot \rangle$ defined by $\langle X, Y \rangle = x \cdot y - x_0 y_0$ for vectors $X = (x, x_0), Y = (y, y_0)$. With regard to this metric, vectors in $\mathbf{R}^{n,1}$ can be divided into spacelike, timelike and null-vectors depending on whether they satisfy $\langle X, X \rangle > 0$, $\langle X, X \rangle < 0$ or $\langle X, X \rangle = 0$. The timelike vectors can be divided further in the natural way into future and past directed vectors.

A hypersurface $M \subset \mathbf{R}^{n,1}$ is called spacelike if it admits an everywhere timelike normal field which we assume to be future directed and to satisfy the condition $\langle \nu, \nu \rangle = -1$. Note that the metric on M induced from $\mathbf{R}^{n,1}$ is Riemannian. Spacelike hypersurfaces can locally be expressed as graphs of functions $u : \Omega \to \mathbf{R}$ satisfying $|Du(x)| < 1$ for all $x \in \Omega$ where Ω is an open subset of $\mathbf{R}^n$. In particular, a spacelike hypersurface satisfies the inequality $|u(x) - u(y)| < |x - y|$ for all $x, y \in \Omega$. In this talk we will concentrate on so-called entire graphs i.e. we will assume $\Omega = \mathrm{R}^n$ unless otherwise stated.