Chaotic Vibrations of the Infinite Dimensional Harmonic Oscillator Due to a Self-Excitation Boundary Condition

Part I: Controlled Hysteresis

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§1 Introduction

Consider the motion of a vibrating string whose displacement $w(x,t)$ at location x at time t satisfies

$$\frac{\partial^2 w}{\partial t^2} - \frac{\partial^2 w}{\partial x^2} = 0, \quad 0 < x < 1, \quad t > 0 \tag{1.1}$$

At the left end $x = 0$, assume the string is fixed:

$$w(0,t) = 0, \quad t > 0 \tag{1.2}$$

At the right end $x = 1$, some force $f(t)$ is acting on the string:

$$w_x(1,t) = f(t), \quad t > 0$$

This force $f(t)$ is assumed to be of the nonlinear velocity feedback type: $f(t) = \alpha w_t(1,t) - \beta w_t^3(1,t)$, $t > 0$, yielding

$$w_x(1,t) = \alpha w_t(1,t) - \beta w_t(1,t)^3, \quad \alpha, \beta > 0. \tag{1.3}$$