STRUCTUAL INEQUALITIES METHOD FOR UNIQUENESS THEOREMS FOR THE MINIMAL SURFACE EQUATION

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1. Existence theorems for elliptic equations

Consider the second order elliptic equations in divergence form:

$$Qu = \mathrm{div}A(x,u,Du) + B(x,u,Du) = 0$$

where $Du = \langle D_1u, ..., D_nu\rangle = \langle \frac{\partial u}{\partial x_1}, ..., \frac{\partial u}{\partial x_n}\rangle$, $A = \langle A_1(x,u,Du), ..., A_n(x,u,Du)\rangle$ and $(A(x,u,Du) - A(x,v,Dv)) \cdot (Du - Dv) > 0$ for every $u, v \in C^1$, $Du \neq Dv$. And consider the Dirichlet problem

$$(1) \qquad \begin{cases} Qu = \mathrm{div}A(x,u,Du) + B(x,u,Du) = 0 & \text{in } \Omega \\ u = \varphi & \text{on } \partial\Omega. \end{cases}$$

It is well-known that the solvability of (1) depends on the structual conditions A, B and the geometric properties of Ω. For example, consider the operator $Qu = \partial_i(a^{ij}u_j) + B$ which is uniformly elliptic. Then the Dirichlet problem (1) is solvable for any bounded smooth domain with continuous boundary value φ.

Now, consider the minimal surface equation $\mathrm{div}Tu = 0$ in Ω where Ω is a bounded smooth domain. Then the Dirichlet problem $\mathrm{div}Tu = 0$ in $\Omega, u = \varphi$ on $\partial\Omega$ is solvable for

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