SOME SLIGHTLY SUBCRITICAL OR SLIGHTLY SUPERCRITICAL PROBLEMS

E.N. DANCER

School of Mathematics and Statistics
University of Sydney
N.S.W. 2006, Australia

The main purpose of the present article is to discuss the existence of positive solutions of the exterior problem

$$\begin{aligned} -\Delta u &= u^{p^*-\epsilon} \text{ in } R^m \backslash \Omega \\ u &= 0 \text{ on } \partial\Omega \\ u &> 0 \text{ on } R^m \backslash \Omega \end{aligned} \tag{1}$$

where $p^* = (m+2)(m-2)^{-1}$ and $m \geq 3$.

Our main result is the following.

Theorem 1. *Assume that $m = 3, 4$ or $6, \Omega$ is a bounded open set in R^m with smooth boundary such that $R^m \backslash \Omega$ is connected and the reduced homology $\widetilde{H}_*(\Omega, Z_2)$ is non-trivial. Then for ϵ small and positive, (1) has a solution*

Remarks 1. The condition on m is only needed for technical reasons and should not be necessary. It is only needed to ensure that the solutions of (1) with $\epsilon = 0$ have a good local structure. In particular, this holds if the solutions of (1) with $\epsilon = 0$ are isolated (for an appropriate norm).

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