NPM planar algebras and the Guionnet-Jones-Shlyakhtenko construction

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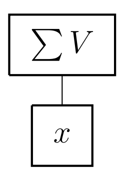
1 Introduction

In [Jon83] Jones initiated the study of modern subfactor theory by defining the index of a subfactor. A stronger combinatorial invariant, called the standard invariant, was later developed and aximoitized by Ocneanu's paragroups [Ocn88], Popa's λ-lattices [Pop95], and Jones' planar algebras [Jon].

Given λ-lattice, Popa constructed a II_1-subfactor whose standard invariant is exactly the given λ-lattice [Pop95]. Later, in in work with Shlyakhtenko, it was shown that the factors in Popa's construction can be made to be isomorphic to the free group factor on infinitely many generators, $L(\mathbb{F}_\infty)$ [PS03]. Guionnet, Jones, and Shlyakhtenko gave a diagrammatic (planar algebraic) proof of Popa's result [GJS10]. In the finite depth case, they showed that the factors involved in the construction are interpolated free group factors [GJS11]. The author later showed that in the infinite depth case, the factors involved in the construction are all isomorphic to $L(\mathbb{F}_\infty)$ [Har13].

This article initially appeared as the final chapter of the author's graduate thesis, and is based on a problem posed by Vaughan Jones. The problem is as follows: Given a subfactor planar algebra, $\mathcal{Q}$, one can consider the algebras $\mathrm{Gr}_k^{\pm}(\mathcal{Q})$ as defined in [GJS10], and place the following "toy potential" on $\mathcal{Q}$:

$\mathrm{tr}(x) =$ 

where V is a rotationally invariant set of elements in $\mathcal{Q}$. If one is fortunate, tr is positive definite on $\mathcal{Q}$ and left multiplication is bounded on $L^2(\mathrm{Gr}(\mathcal{Q}))$. To this end, it is an interesting problem to study the von Neumann algebras, $N_k^{\pm}(= \mathrm{Gr}_k^{\pm}(\mathcal{Q})'')$ associated to $\mathcal{Q}$ and V.

The case that will be considered here is the case where $\mathcal{Q}$ is the standard invariant for a subfactor $N \subset M$ that contains an intermediate subfactor, P. As such, it follows that $\mathcal{Q}$ contains the Fuss Catalan planar algebra as a sub planar algebra [BJ97]. Therefore, we can consider the following potential on $\mathrm{Gr}_0^+(\mathcal{Q})$:

$\mathrm{tr}(x) =$