APPENDIX A

A GENERAL REGULARITY THEOREM

We here prove a useful general regularity theorem, which is essentially an abstraction of the "dimension reducing" argument of Federer [FH2]. There are a number of important applications of this general theorem in the text.

Let $P \geq n \geq 2$ and let F be a collection of functions $\phi = (\phi^1, \ldots, \phi^Q) : \mathbb{R}^P \to \mathbb{R}^Q$ ($Q = 1$ is an important case) such that each ϕ^j is locally H^n-integrable on $\mathbb{R}^P$. For $\phi \in F$, $y \in \mathbb{R}^P$ and $\lambda > 0$ we let $\phi_{y,\lambda}$ be defined by

$$\phi_{y,\lambda}(x) = \phi(y+\lambda x) \text{ , } x \in \mathbb{R}^P \text{ .}$$

Also, for $\phi \in F$ and a given sequence $\{\phi_k\} \subset F$ we write $\phi_k \rightharpoonup \phi$ if $\int \phi_k f \, dH^n \to \int \phi f \, dH^n$ (in $\mathbb{R}^Q$) for each given $f \in C_c^0(\mathbb{R}^P)$.

We subsequently make the following 3 special assumptions concerning F:

A.1 (Closure under appropriate scaling and translation): *If* $|y| \leq 1-\lambda$, $0 < \lambda < 1$, *and if* $\phi \in F$, *then* $\phi_{y,\lambda} \in F$.

A.2 (Existence of homogeneous degree zero "tangent functions"): *If* $|y| < 1$, *if* $\{\lambda_k\} \downarrow 0$ *and if* $\phi \in F$, *then there is a subsequence* $\{\lambda_{k'}\}$ *and* $\psi \in F$ *such that* $\phi_{y,\lambda_{k'}} \rightharpoonup \psi$ *and* $\psi_{0,\lambda} = \psi$ *for each* $\lambda > 0$.

A.3 ("Singular set" hypotheses): *We assume there is a map*

$$\text{sing} : F \to C \text{ (} = \text{set of closed subsets of } \mathbb{R}^P)$$

such that

(1) $\text{sing}\, \phi = \emptyset$ *if* $\phi \in F$ *is a constant multiple of the characteristic function of an* n*-dimensional subspace of* $\mathbb{R}^P$,