CHAPTER 8

THEORY OF GENERAL VARIFOLDS

Here we describe the theory of general varifolds, essentially following W.K. Allard [AW1].

General varifolds in U (U open in $\mathbb{R}^{n+k}$) are simply Radon measures on $G_n(U) = \{(x,S) : x \in U$ and S is an n-dimensional subspace of $\mathbb{R}^{n+k}\}$. One basic motivating point for our interest in such objects is described as follows:

Suppose $\{T_j\}$ is a sequence of integer multiplicity currents (see §27) such that the corresponding integer multiplicity varifolds (as in Chapter 4) are stationary in U (U open in $\mathbb{R}^{n+k}$), and suppose $\partial T_j = 0$ and there is a mass bound $\sup_{j\geq 1} \underline{M}_W(T_j) < \infty \;\; \forall\, W \subset\subset U$. By the compactness theorem 27.3 we can assert that $T_{j'} \rightharpoonup T$ for some integer multiplicity T. However it is *not* clear that T is stationary; the chief difficulty is that it is *not* generally true that the corresponding sequence of measures $\mu_{T_{j'}}$ converge to μ_T. Indeed if $\mu_{T_{j'}}$ converges to μ_T (as they would by 34.5 in case the T_j are minimizing in U) then it is not hard to prove that T is stationary in U. This leads one to consider measure theoretic convergence rather than weak convergence of the currents. However if we take a limit (in the sense of Radon measures) of some sub-sequence $\{\mu_{T_{j'}}\}$ of the $\{\mu_{T_j}\}$ then we get merely an abstract Radon measure on U, and first variation of this does not make sense.

To resolve these difficulties, we associate with each T_j a Radon measure V_j on the Grassmaniann $G_n(U)$ ($G_n(U)$ is naturally equipped with a suitable metric - see below); V_j is in fact defined by