CHAPTER 7

AREA MINIMIZING CURRENTS

This chapter provides an introduction to the theory of area minimizing currents. In the first section (§33) of the chapter we derive some basic preliminary properties, and in particular we discuss the fact that the integer multiplicity varifold corresponding to a minimizing current is stable (and indeed minimizing in a certain sense). In §34 there are some existence and compactness results, including the important theorem that if $\{T_j\}$ is a sequence of minimizing currents in U with $\sup_{j\geq 1}(\underline{\underline{M}}_W(T_j) + \underline{\underline{M}}_W(\partial T_j)) < \infty \quad \forall\, W \subset\subset U$, and if $T_j \rightharpoonup T \in \mathcal{D}_n(U)$, then T is also minimizing in U and the corresponding varifolds converge in the measure theoretic sense of §15. This enables us to discuss tangent cones and densities in §35, and in particular make some regularity statements for minimizing currents in §36. Finally, in §37 we develop the standard codimension 1 regularity theory, due originally to De Giorgi [DG], Fleming [FW], Almgren [A4], J. Simons [SJ] and Federer [FH2].

§33. BASIC CONCEPTS

Suppose A is any subset of $\mathbb{R}^{n+k}$, $A \subset U$, U open in $\mathbb{R}^{n+k}$, and $T \in \mathcal{D}_n(U)$ an integer multiplicity current.

33.1 DEFINITION We say that T is minimizing in A if

$$\underline{\underline{M}}_W(T) \leq \underline{\underline{M}}_W(S)$$

whenever $W \subset\subset U$, $\partial S = \partial T$ (in U) and $\mathrm{spt}(S-T)$ is a compact subset of $A \cap W$.