CHAPTER 6

CURRENTS

This chapter provides an introduction to the basic theory of currents, with particular emphasis on integer multiplicity rectifiable n-currents (briefly called integer multiplicity currents), which are essentially just integer n-varifolds equipped with an orientation.* The concept of such currents was introduced in the historic paper [FF] of Federer and Fleming; their advantage is that they are at once able to be represented as "generalized surfaces" (in terms of a countably n-rectifiable set with an integer multiplicity) and at the same time have nice compactness properties (see 27.3 below).

§25. PRELIMINARIES: VECTORS, CO-VECTORS, AND FORMS

$e_1, \ldots, e_P$ denote the standard orthonormal basis for $\mathbb{R}^P$ and $\omega^1, \ldots, \omega^P$ the dual basis for the dual space $\Lambda^1(\mathbb{R}^P)$ of $\mathbb{R}^P$. $\Lambda_n(\mathbb{R}^P), \Lambda^n(\mathbb{R}^P)$ denote the spaces of n-vectors and n-covectors respectively. Thus $v \in \Lambda_n(\mathbb{R}^P)$ can be represented

$$v = \sum_{1 \le i_1 < \ldots < i_n \le P} a_{i_1 \ldots i_n} e_{i_1} \wedge \ldots \wedge e_{i_n}$$

$$= \sum_{\alpha \in I_{n,P}} a_\alpha e_\alpha ,$$

using "multi-index" notation in which $\alpha = (i_1, \ldots, i_n) \in \mathbb{Z}_+^n \equiv \{(j_1, \ldots, j_n)$: each j_ℓ is a positive integer$\}$ and $I_{n,P} = \{\alpha = (i_1, \ldots, i_n) \in \mathbb{Z}_+^n$: $1 \le i_1 < \ldots < i_n \le P\}$. Similarly any $w \in \Lambda^n(\mathbb{R}^P)$ can be represented as

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* These are precisely the currents called *locally rectifiable* in the literature (see [FF], [FH1]); we have adopted the present terminology both because it seems more natural and also because it is consistent with the varifold terminology of Allard (see Chapter 4, Chapter 8).