CHAPTER 4

THEORY OF RECTIFIABLE n-VARIFOLDS

Let M be a countably n-rectifiable, $\mathcal{H}^n$-measurable subset of $\mathbb{R}^{n+k}$, and let θ be a positive locally $\mathcal{H}^n$-integrable function on M. Corresponding to such a pair (M, θ) we define the rectifiable n-varifold $\underline{\underline{v}}(M,\theta)$ to be simply the equivalence class of all pairs $(\tilde{M},\tilde{\theta})$, where $\tilde{M}$ is countably n-rectifiable with $\mathcal{H}^n((M\sim\tilde{M})\cup(\tilde{M}\sim M)) = 0$ and where $\tilde{\theta} = \theta$ $\mathcal{H}^n$-a.e. on $M \cap \tilde{M}$.* θ is called the *multiplicity function* of $\underline{\underline{v}}(M,\theta)$. $\underline{\underline{v}}(M,\theta)$ is called an integer multiplicity rectifiable n-varifold (more briefly, an *integer* n-*varifold*) if the multiplicity function is integer-valued $\mathcal{H}^n$-a.e.

In this chapter and in Chapter 5 we develop the theory of general n-rectifiable varifolds, particularly concentrating on *stationary* (see §16) rectifiable n-varifolds, which generalize the notion of classical minimal submanifolds of $\mathbb{R}^{n+k}$. The key section is §17, in which we obtain the monotonicity formulae; much of the subsequent theory is based on these and closely related formulae.

§15. BASIC DEFINITIONS AND PROPERTIES

Associated to a rectifiable n-varifold $V = \underline{\underline{v}}(M,\theta)$ (as described above) there is a Radon measure μ_V (called the *weight measure* of V) defined by

15.1 $$\mu_V = \mathcal{H}^n \llcorner \theta ,$$

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* We shall see later that this is essentially equivalent to Allard's ([AW1]) notion of n-dimensional rectifiable varifold. In case $M \subset U$, U open in $\mathbb{R}^{n+k}$ and θ is locally $\mathcal{H}^n$-integrable in U, we say $V = \underline{\underline{v}}(M,\theta)$ (as defined above) is *a rectifiable* n-*varifold in* U.