CHAPTER 3

COUNTABLY n-RECTIFIABLE SETS

The countably n-rectifiable sets, the theory of which we develop in this chapter, provide the appropriate notion of "generalized surface"; they are the sets on which rectifiable currents and varifolds live (see later).

In the first section of this chapter we give some basic definitions, and prove the important result that countably n-rectifiable sets are essentially characterized by the property of having a suitable "approximate tangent space" almost everywhere.

In later sections we show that the area and co-area formula (see §§8,10 of Chapter 2) extend naturally to the case when M is merely countably n-rectifiable rather than a C^1 submanifold, we make a brief discussion of Federer's structure theorem (for the proof we refer to [FH1] or [RM]), and finally we discuss sets of finite perimeter, which play an important role in later developments.

§11. BASIC NOTIONS, TANGENT PROPERTIES

Firstly, a set $M \subset \mathbb{R}^{n+k}$ is said to be countably n-rectifiable if $M \subset M_0 \cup (\bigcup_{j=1}^{\infty} F_j(\mathbb{R}^n))$, where $\mathcal{H}^n(M_0) = 0$ and $F_j : \mathbb{R}^n \to \mathbb{R}^{n+k}$ are Lipschitz functions for $j = 1,2,\ldots$ * . Notice that by the extension theorem 5.1 this is equivalent to saying

$$M = M_0 \cup (\bigcup_{j=1}^{\infty} F_j(A_j))$$

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* Notice that this differs slightly from the terminology of [FH1] in that we allow the set M_0 of $\mathcal{H}^n$-measure zero.