CHAPTER 1

PRELIMINARY MEASURE THEORY

In this chapter we briefly review the basic theory of outer measure (with Caratheodory's definition of measurability). Hausdorff measure is discussed, including the main results concerning n-dimensional densities and the way in which they relate more general measures to Hausdorff measures. The final section of the chapter gives the basic theory of Radon measures (including the Riesz representation theorem and the differentiation theory).

Throughout the chapter X will denote a metric space with metric d. In the last section X satisfies the additional requirements of being locally compact and separable.

§1. BASIC NOTIONS

Recall that an *outer measure* (henceforth simply called a *measure*) on X is a *monotone subadditive* function $\mu : 2^X \to [0,\infty]$ with $\mu(\phi) = 0$. Thus $\mu(\phi) = 0$ and

$$\mu(A) \leq \sum_{j=1}^{\infty} \mu(A_j) \quad \text{whenever} \quad A \subset \bigcup_{j=1}^{\infty} A_j$$

with $A, A_1, A_2, \ldots$ any countable collection of subsets of X. Of course this in particular implies $\mu(A) \leq \mu(B)$ whenever $A \subset B$.

We adopt Caratheodory's notion of *measurability* :

A subset $A \subset X$ is said to be μ-measurable if