Results and Conjectures in Mathematical Relativity

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In 1916 [1] Albert Einstein presented his well-known equation

$$R_{\alpha\beta} - \frac{1}{2} R\, g_{\alpha\beta} = 8\pi\, T_{\alpha\beta} ,$$

relating the geometry of space and time, modelled by a 4-dimensional manifold with Lorentz metric $g_{\alpha\beta}$ and Ricci curvature $R_{\alpha\beta}$, to the physical matter distribution, modelled by the stress-energy tensor $T_{\alpha\beta}$. This was a truly revolutionary theory in that it lead to major changes, both philosophical and scientific, in the way we view our world.

The effect on physics has perhaps been the most noticeable. Quite apart from its surprising predictions about the large scale geometry of the universe, general relativity introduced the idea of a "Theory Of Everything", now a Holy Grail of theoretical physics, and it showed that differential geometry is the natural language of physics. The success of the Yang–Mills–Higgs model of the electroweak interaction, and of the intensive work in areas such as string theory and supergravity, shows clearly that these ideas have had some impact on physics.

The effect of Einstein's theory on mathematics, especially differential geometry, was just as pronounced. By emphasising the physical relevance of such questions as the relation between curvature, geodesics and geometry, and the nature of Maxwell's equations on a manifold, the outlines of the subject we now recognise as differential geometry were laid.