THE GEOMETRY AND PHYSICS OF KNOTS.

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1. LINKING NUMBERS AND FUNCTIONAL INTEGRALS

1.1 INTRODUCTION

The aim of these lectures is to present a new approach to the Jones polynomial invariants of knots (Annals of Math. 1988) due to Witten ("Jones polynomial and quantum field theory" to appear in Proceedings IAMP Swansea 1988). They represent a very abbreviated version in which many subtle points have been omitted or only alluded to.

1.2 KNOTS AND LINKS IN $\mathbf{R}^3$

A knot is just an oriented closed connected smooth curve in $\mathbf{R}^3$. A general curve with possibly many components is referred to as a link. Knots may also be considered as embedded in S^3 or more generally in an arbitrary (compact, oriented) three dimensional manifold M^3.

The main problem is to classify knots by suitable invariants. The earliest attempt was the introduction by Alexander (1928) of a one variable polynomial knot invariant with integral coefficients. The Alexander polynomial is not a complete invariant for knots but is useful and readily computable. Moreover it can be constructed from standard techniques of algebraic topology (homology of a covering branched over the knot). One defect of the Alexander polynomial is that it fails to distinguish 'chirality', that is a knot and its mirror image have the same polynomial.

The Jones polynomial (1984) $V(q)$ is a finite Laurent series in q with the following properties.

1) It is chiral giving different values for example to the left handed and right handed trefoil knots.
2) It is associated with the Lie group $SU(2)$ and there are other polynomial invariants