STEFAN PROBLEMS

John van der Hoek

§0. Introduction

In 1899 STEFAN [1] posed the following problem : A heat conducting material occupies the space $-\infty < x < \infty$. Initially ($t = 0$), the liquid phase occupies $-\infty < x < 0$ at temperature $T_1 > 0$ and the solid phase occupies $0 < x < \infty$ at temperature $T_2 < 0$. It is required to determine temperatures $u^1(x,t)$ and $u^2(x,t)$ at position x and time t of the liquid and solid phases, respectively, and the position $x = s(t)$ of the free or moving boundary between the phases as a function of time (t). Stefan showed that the following thermal balance operates between the two phases :

$$(1) \qquad -\lambda\rho\,\frac{ds}{dt} = \left(\kappa_1\,\frac{\partial u^1}{\partial x} - \kappa_2\,\frac{\partial u^2}{\partial x}\right)\Big|_{x=s(t)}$$

where λ = latent heat, ρ = density of the material in its original phase state, κ_1 and κ_2 are coefficients of conductivity corresponding to the liquid and solid phases, respectively. Condition (1) will be referred to as the Stefan free boundary condition. This relationship holds, for example, at the interface between water and ice in the process of melting ice (see RUBINSTEIN [2]). The full formulation of Stefan's problem would also include in addition to (1),

$$(2) \qquad c_1\,\frac{\partial u^1}{\partial t} = \frac{\partial}{\partial x}\left(\kappa_1\,\frac{\partial u^1}{\partial x}\right), \; s(t) < x < \infty, \; t > 0$$

$$(3) \qquad c_2\,\frac{\partial u^2}{\partial t} = \frac{\partial}{\partial x}\left(\kappa_2\,\frac{\partial u^2}{\partial x}\right), \; -\infty < x < s(t), \; t > 0$$