MAXIMUM PRINCIPLE FOR NON-LINEAR DEGENERATE INEQUALITIES OF PARABOLIC TYPE

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In recent years the maximum principle was extended to degenerate elliptic parabolic equations, and has been studied by several authors, for example in [2], [3], [4], [5], [6], [8], [9], [10]. In this paper we consider a differential inequality

$$(1) \qquad \alpha(t,x)u_t - f(t,x,u(t,x),Du(t,x),D^2u(t,x)) \leq$$
$$\alpha(t,x)v_t - f(t,x,v(t,x),Dv(t,x),D^2u(t,x))$$

in $Q = (0,T] \times \Omega$, where Ω is an open and bounded set in R^n, and $\alpha(t,x) \geqq 0$ in Q. Du denotes the gradient of u with respect to x, D^2u is the Hessian matrix of the second order derivatives (also with respect to the variable x). $f(t,x,u,p,r)$ is assumed to be defined for $(t,x) \in \phi$, $u \in R$, $p \in R^n$ and $r \in R^{n^2}$.

The main assumptions are that (i) f is weakly parabolic in sense of Besala (see [1]) (ii) f is decreasing with respect to u, (iii) f is Lipschitz with respect to p and r and (iv) there exists a positive constant h and non-negative function γ such that

$$(2) \qquad \alpha(t,x) + \gamma(t,x) \geq h$$

for all $(t,x) \in Q$. If u and w are regular, (for definition see [7]) satisfy (1) and $u - v$ has a non-negative maximum on $\bar{Q}$ then this maximum is attained at some point of the parabolic boundary of Q. Simple examples