ON ISOLATED SINGULARITIES OF MINIMAL SURFACES

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We here want to give a brief discussion of some questions related to minimal surfaces with isolated singularities; the many questions related to minimal surfaces with more complicated singular sets are not considered.

We first make our terminology precise. For simplicity of exposition we discuss embedded submanifolds of Euclidean space - making the necessary comments about the more general Riemannian setting at appropriate points.

M will denote a smooth n-dimensional embedded submanifold of $\mathbb{R}^{n+k}$, $n \geq 2$, $k \geq 1$, where we always use the term "embedded" to mean locally properly embedded. Thus for each $y \in M$ there is an open ball $B_\rho(y)$ with centre y and radius $\rho > 0$, and a C^2 diffeomorphism ψ of $B_\rho(y)$ onto $B_\rho(0)$ such that $\psi\left(M \cap B_\rho(y)\right) = \mathbb{R}^n \cap B_\rho(0)$. Here and subsequently we identify $\mathbb{R}^n$ with the subspace of $\mathbb{R}^{n+k}$ consisting of all points $\left(x^1, \ldots, x^{n+k}\right)$ such that $x^j = 0 \quad \forall j = n+1, \ldots, n+k$.

M is said to be *minimal* if the mean curvature of M is identically zero. As is well-known, this condition is equivalent to the *local area minimizing property*: for each $y \in M$ there is some open ball $B_\rho(y) \subset \mathbb{R}^{n+k}$ such that