REGULARITY AT THE BOUNDARY AND REMOVABLE SINGULARITIES FOR SOLUTIONS OF QUASILINEAR PARABOLIC EQUATIONS

William P. Ziemer

1. INTRODUCTION

The purpose of this note is to describe recent results concerning removable singularities and behavior of weak solutions of quasilinear parabolic equations of the second order at the boundary of an arbitrary domain. Specifically, we investigate the local behaviour of weak solutions of equations of the form

$$u_t = \operatorname{div} A(x,t,u,u_x) + B(x,t,u,u_x) \tag{1}$$

where A and B are, respectively, vector and scalar valued Borel functions defined on $\Omega \times R^1 \times R^n$, where Ω is arbitrary open subset of $R^{n+1}(x,t)$. The functions A and B are required to satisfy the following structure conditions:

$$|A(x,t,u,w)| \le a_0|w|^{p-1} + (a_1|u|)^{p-1} + a_2^{p-1}$$

$$|B(x,t,u,w)| \le b_0|w|^p + b_1|w|^{p-1} + (b_2|u|)^{p-1} + b_3^{p-1} \tag{2}$$

$$A(x,t,u,w) \cdot w \ge |w|^p - (c_1|u|)^p - c_2^p .$$

Here, $p > 1$, $a_0 > 0$, $b_0 > 0$ and the remaining coefficients are non-negative functions of (x,t) that are required to belong to specified Lebesgue classes. For the purposes of this exposition, we will simply require a_1^p, a_2^p, b_1^p, b_2^{p-1}, b_3^{p-1}, c_1^p and c_2^p to lie in $L^q(\Omega)$ where $\frac{n}{pq} + \frac{1}{q} < 1$.