ELLIPTIC EQUATIONS IN NON-DIVERGENCE FORM

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This article is concerned with certain local estimates of Harnack and Hölder type that have been established recently for linear and non-linear elliptic partial differential equations in the works of Krylov and Safonov, [10], [11], Trudinger [19], [20], and Evans [7], [8]. Crucial to the derivation of these results is a maximum principle that was discovered about twenty years ago by Aleksandrov [2] and Bakelman [5]. Since the Aleksandrov-Bakelman maximum principle has received only scant attention in expository monographs such as [9] we will also supply its proof below.

Let Ω be a bounded domain in Euclidean n space, $\mathbb{R}^n$ and let $A = [a^{ij}]$ be a measurable, real $n\times n$ symmetric matrix valued function on Ω. We assume that A is positive in Ω so that the partial differential operator L, given by

$$Lu = a^{ij}D_{ij}u \tag{1}$$

for $u \in C^2(\Omega)$ is *elliptic* in Ω. (As is customary we adopt the summation convention that repeated indices indicate summation from 1