19. NUMERICAL EXAMPLES

In this section we report the results of our numerical calculations and illustrate the use of the algorithms given in Section 17.

We consider the space $X = C([a,b])$ of all complex-valued continuous functions on the interval $[a,b]$. Given a positive integer M, let $t_1^{(M)},\dots,t_M^{(M)}$ be points in $[0,1]$ and let

$$(19.1) \qquad \pi_M x = \sum_{i=1}^{M} \langle x, f_i^* \rangle f_i \ ,$$

where $\langle x, f_i^* \rangle = x(t_i^{(M)})$, and $f_i \in C([0,1])$ is such that $f_i(t_j^{(M)}) = \delta_{i,j}$, $i,j = 1,\dots,M$. An element x of $C([a,b])$ is discretized by $\pi_M x$.

Let T be a Fredholm integral operator on $C([a,b])$ given by

$$Tx(s) = \int_a^b k(s,t)x(t)dt \ , \quad s \in [a,b] \ , \quad x \in C([0,1]) \ ,$$

where k is a continuous complex-valued function on $[a,b] \times [a,b]$. Note that T is a compact operator on $C([0,1])$.

Given a convergent quadrature formula (cf.(16.5))

$$\sum_{j=1}^{M} w_j^{(M)} x(t_j^{(M)}) \ , \quad x \in C([a,b]) \ ,$$

with nodes at $t_j^{(M)}$, $j = 1,\dots,M$, we replace the operator T by its Nyström approximation

$$(19.2) \quad \tilde{T}x(s) = \sum_{j=1}^{M} w_j^{(M)} k(s,t_j^{(M)}) x(t_j^{(M)}) \ , \quad s \in [a,b] \ , \quad x \in C([a,b]) \ ,$$

which is easier to handle numerically. From now on we do not make any distinction between T and $\tilde{T}$.