11. ERROR BOUNDS FOR ITERATIVE REFINEMENTS

A customary way for approximating eigenelements λ, φ of $T \in BL(X)$ is to consider a nearby simpler operator T_0, solve the eigenvalue problem

$$T_0\varphi_0 = \lambda_0\varphi_0 \ , \ 0 \neq \varphi_0 \in X \ , \ \lambda_0 \in \mathbb{C} \ ,$$

and refine the eigenelements λ_0, φ_0 of T_0 successively to obtain approximations of λ, φ .

In this section we develop some refinement schemes of this type when λ_0 is *simple*. We also show that two main iteration schemes lead to a *simple* eigenvalue λ of T ; a region of isolation for λ from the rest of $\sigma(T)$ is also found. We conclude this section with a discussion of the power method, the inverse iteration and the Rayleigh quotient iteration.

We shall assume throughout this section that λ_0 is a simple eigenvalue of $T_0 \in BL(X)$, and φ_0 (resp., φ_0^*) is an eigenvector of T_0 (resp., T_0^*) corresponding to λ_0 (resp., $\bar{\lambda}_0$) such that $\langle \varphi_0, \varphi_0^* \rangle = 1$. Let P_0 and S_0 denote, as usual, the spectral projection and the reduced resolvent associated with T_0 and λ_0 , respectively. We let $V_0 = T - T_0$, so that $T = T_0 + V_0$, and seek an eigenvector φ of T which satisfies the same condition : $\langle \varphi, \varphi_0^* \rangle = 1$.

We recall the notations introduced in (10.16):

$$\eta_0 = \|V_0\varphi_0\| \ , \ p_0 = \|\varphi_0^*\| \ , \ s_0 = \|S_0\| \ ,$$
$$\alpha_0 = \|V_0S_0\| \ , \ \gamma_0 = \max\{\eta_0 p_0 s_0 \ , \ \alpha_0\} \ .$$

Note that if $\gamma_0 = 0$, then $\eta_0 = 0 = \alpha_0$, so that $V_0P_0 = 0 = V_0S_0$; this implies $V_0 = 0$. We discard this trivial case.