8. SPECTRUM OF THE ADJOINT OPERATOR

In this section we investigate resolvent operators, spectral projections, reduced resolvents and quasi-nilpotent operators associated with the adjoint T^* of an operator $T \in BL(X)$. The underlying story behind these results is that the operation of taking an adjoint of an operator in $BL(X)$ is like the operation of taking the complex conjugate of a complex number. We shall see that points in the discrete spectrum of T^* correspond to points in the discrete spectrum of T; thus the situation here is analogous to the finite dimensional case. The concept of a Rayleigh quotient is introduced and used to obtain estimates for an eigenvalue. We conclude this section by proving the spectral theorem for compact normal operators, and by pointing out some special results for self-adjoint operators.

THEOREM 8.1 Let $T \in BL(X)$. Then

(a) $$\rho(T^*) = \{\bar{z} : z \in \rho(T)\} ,$$

(8.1) $$\sigma(T^*) = \{\bar{\lambda} : \lambda \in \sigma(T)\} ,$$

and for $z \in \rho(T)$, we have

(8.2) $$[R(T,z)]^* = R(T^*,\bar{z}) .$$

(b) Let Γ be a (positively oriented simple rectifiable closed) curve in $\rho(T)$, and let $\bar{\Gamma}$ be the conjugate curve. Then

(8.3) $$[P_\Gamma(T)]^* = P_{\bar{\Gamma}}(T^*) ,$$

(8.4) $$[S_\Gamma(T,z)]^* = S_{\bar{\Gamma}}(T^*,\bar{z}) \quad \text{for} \quad z \notin \Gamma ,$$

(8.5) $$[D_\Gamma(T,z)]^* = D_{\bar{\Gamma}}(T^*,\bar{z}) \quad \text{for} \quad z \in \mathbb{C} .$$