1. ADJOINT CONSIDERATIONS

A useful way of studying a complex Banach space X and a bounded linear operator T on X is to consider the adjoint space

$$X^* = \{x^* : X \to \mathbb{C} \ , \ x^* \ \text{is conjugate linear and continuous}\}$$

of X and the *adjoint operator* T^* associated with T. In this section we develop these concepts. This is done in such a way as to make the well-known Hilbert space situation a particular case of our development.

For $x^* \in X^*$ and $x \in X$, we denote the value of x^* at x by

$$\langle x^*, x\rangle \ .$$

Then we easily see that for x^* and y^* in X^*, x and y in X and $t \in \mathbb{C}$,

$$\begin{aligned} \langle x^*, x+y\rangle &= \langle x^*, x\rangle + \langle x^*, y\rangle \ , \\ \langle x^*, tx\rangle &= \bar{t}\langle x^*, x\rangle \ , \\ \langle x^*+y^*, x\rangle &= \langle x^*, x\rangle + \langle y^*, x\rangle \ , \\ \langle tx^*, x\rangle &= t\langle x^*, x\rangle \ . \end{aligned} \tag{1.1}$$

We say that $\langle \ , \ \rangle$ is the scalar product on $X^* \times X$. For the sake of convenience, we introduce the following notation:

$$\langle x, x^*\rangle = \overline{\langle x^*, x\rangle} \ , \quad x \ \text{in} \ X \ \text{and} \ x^* \ \text{in} \ X^* \ . \tag{1.2}$$

For x^* in X^*, let

$$\|x^*\| = \sup\{\,|\langle x^*, x\rangle| : x \ \text{in} \ X \ , \ \|x\| \le 1\} \ .$$