

The Valuation of Contingent Securities.

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In financial economics a contingent security is an instrument that can be traded now, and which returns at some fixed time T in the future an amount that depends on the value at time T of some underlying security (usually a share). Since the price s at time T of the security is effectively a random variable on $[0, \infty]$, the contingent security may be identified with the function $f(s)$ specifying its return. For example, the holder of a call option has the right, but not the obligation, to buy a share at some agreed strike price s_0 at time T ; the associated return function is $f(s) = \max\{0, s - s_0\}$. The problem is to determine the present value (i.e. price) of such a contingent security, or equivalently to identify the pricing functional $V: L \rightarrow \mathfrak{R}$, (L is the set of feasible return functions).

At present option pricing is done via the Black-Scholes formula [4]. The formula is derived by constructing a self-financing portfolio consisting of shares and bonds (a bond is a risk-free security whose return function is a constant) which is adjusted by trading so that it replicates the return of the contingent security. The option must have the same present value as the portfolio, otherwise buying the portfolio and selling the security, or vice versa, would give certain profit now, while the future returns of the two assets would cancel. Under the basic assumptions that the underlying stock price evolves through a geometric Brownian motion and that one can trade continuously in the market, a simple p.d.e. can be derived that describes the evolution of the portfolio, and this can then be integrated backward to determine the present option price. This approach is of great practical use, however its validity is restricted by the strong assumptions made about the market. Our aim is to find alternative characterizations of V .

In a perfect market where all contingent securities could be traded, the prices of securities contingent on a particular underlying security must be consistent if the market is in equilibrium. If not, there would be the possibility of riskless profit taking, which would in turn move prices. These consistency conditions essentially require that the functional V be linear and positive. Thus if L is a subset of any reasonable function space, e.g. L is the positive cone of $LP([0, \infty], \mu)$ for some p and finite measure μ , then V must be a bounded linear functional [6, p. 84].

The problem brought to the workshop arose from an attempt to further characterize the pricing functional through the general equilibrium theory of Arrow and Debreu [1]. Briefly, given a