1.6. Analytic Vectors.

In the previous sections we examined various methods of constructing a contraction semigroup from the resolvent of its generator. Next we analyze the possibility of a direct construction based on an operator extension of the numerical algorithms

$$\exp\{-tx\} = \sum_{n\geq 0} \frac{(-t)^n}{n!} x^n$$

$$= \lim_{n\to\infty} \left(1-\frac{t}{n}x\right)^n .$$

The problem with this new construction is that it is not applicable to all C_0-semigroups, or contraction semigroups, although it is applicable to all C_0-groups. The basic new concept is that of an analytic element.

If H is an operator on a Banach space $\mathcal{B}$ an element $a \in \mathcal{B}$ is defined to be an *(entire) analytic element for* H if

$$a \in \bigcap_{n\geq 1} D(H^n)$$

and the function

$$t \geq 0 \longmapsto \sum_{n\geq 0} \frac{t^n}{n!} \|H^n a\|$$

has a non-zero (infinite) radius of convergence. It is not at all evident that an operator possesses analytic elements but this is indeed the case