1.3. Generators and Semigroups.

Proposition 1.2.1 states necessary conditions for an operator to generate a C_0-semigroup of contractions. Next we examine sufficient conditions and also study the construction of a semigroup from its generator.

The problem of characterizing a generator H is equivalent to the problem of proving existence and uniqueness of global solutions of a differential equation

$$\frac{da_t}{dt} + Ha_t = 0 , \qquad a_t = a$$

for all a in a suitable Banach space $\mathcal{B}$. Formally the solution of the differential equation is

$$a_t = \exp\{-tH\}a$$

and the difficulty is to give an appropriate definition of the exponential. Various algorithms and approximation techniques are of use. For example the algorithm

$$\exp\{-tx\} = \lim_{n\to\infty} (1+tx/n)^{-n}$$

for the numerical exponential can be extended to an operator relation if the (pseudo-) resolvent $(I+\alpha H)^{-1}$ has suitable properties for small positive α.

It should perhaps be emphasized that in applications