1.2. Semigroups and Generators.

Let $\mathcal{B}$ be a complex Banach space and $\mathcal{B}^*$ its dual. We denote elements of $\mathcal{B}$ by a, b, c, ... and elements of $\mathcal{B}^*$ by f, g, h, ... Moreover we use (f, a) to denote the value of f on a and $\|\cdot\|$ to denote the norm on $\mathcal{B}$ and also the dual norm on $\mathcal{B}^*$, i.e.,

$$\|f\| = \sup\{|f(a)| \; ; \; \|a\| \le 1\} .$$

A *semigroup* S on $\mathcal{B}$ is defined to be a family $S \; ; \; t \in \mathbb{R}_+ \mapsto S_t \in \mathcal{L}(\mathcal{B})$ of bounded linear operators on $\mathcal{B}$ which satisfy

1. $S_sS_t = S_{s+t}$, $\quad s, t \ge 0$,

2. $S_0 = I$

   where I denotes the identity operator on $\mathcal{B}$.

This notion of semigroup is not of great interest unless one imposes some further hypothesis of continuity. There are a variety of possible forms of continuity. Let us first consider continuity at the origin.

The strongest possible requirement would be uniform continuity, i.e.,

$$\lim_{t \to 0+} \|S_t - I\| = 0 ,$$

where the operator norm is defined in the usual manner