1.1. Introduction.

Continuous one-parameter semigroups of bounded operators occur in many branches of mathematics, both pure and applied. The calculus of functions of one real variable can be formulated in terms of the translation semigroup, solutions of the equations connected with classical phenomena such as heat propagation are described by semigroups, and one-parameter groups and semigroups also describe the dynamics of quantum mechanical systems. Although semigroups occur in many other areas the development and scope of the general theory covered in this chapter is well illustrated by the foregoing examples. Hence we begin with a brief discussion of each of them.

The semigroup of right translations on $C_0(\mathbb{R})$, the continuous functions over the real line which vanish at infinity, is defined by

$$f \in C_0(\mathbb{R}) \longmapsto S_t f \in C_0(\mathbb{R}) ,$$

where

$$(S_t f)(x) = f(x-t) .$$

Thus one has the semigroup property

$$S_s S_t = S_{s+t} , \qquad s, t \geq 0$$

and

$$S_0 = I$$