CHAPTER 4

REGULARITY OF WEAKLY HARMONIC MAPS

Regularity, existence, and uniqueness of solutions of the Dirichlet problem, if the image is contained in a convex ball

4.1 THE CONCEPT OF WEAK SOLUTIONS

We first want to discuss the concept of stationary points of the energy integral or of weak solutions of the corresponding Euler-Lagrange equations. In the present chapter, the image Y will always be covered by a single coordinate chart so that we can define the Sobolev space $W_2^1(\Omega, Y)$ unambiguously with the help of this chart, without having to use the Nash embedding theorem as in 1.3.

Ω will be an open bounded set in some Riemannian manifold with boundary $\partial\Omega$.

In the sequel, we shall use some of the notations of [EL4].

If $u \in W_2^1(\Omega, Y)$, then du is an almost everywhere on Ω defined 1-form with values in $u^{-1}TY$. The energy of u is

$$E(u) = \frac{1}{2} \int_\Omega \langle du, du \rangle \, d\Omega ,$$

where the scalar product is taken in $T^*\Omega \otimes u^{-1}TY$.

We let $\phi \in C_0(\bar{\Omega}, u^{-1}TY)$ be a section along u which vanishes on $\partial\Omega$. This means $\phi(x) \in T_{u(x)}Y$. We want to construct a variation of u with tangent field ϕ .

Since we assume that Y is covered by a single coordinate chart, we can simply represent everything in those coordinates and denote the representations in these coordinates by $\tilde{}$ and define