THE ITERATED GALERKIN METHOD FOR INTEGRAL EQUATIONS OF THE SECOND KIND

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1. INTRODUCTION

Consider the integral equation of the second kind

$$y(t) = f(t) + \int_{\Omega} k(t,s)y(s)d\sigma(s) , \quad t \in \Omega , \tag{1.1}$$

where Ω is either a bounded domain in $\mathbb{R}^d$ with a locally Lipschitz boundary or the smooth d-dimensional boundary of a bounded domain in $\mathbb{R}^{d+1}$, and $d\sigma(s)$ is the element of volume or surface area, as appropriate. Writing the equation as

$$y = f + Ky , \tag{1.2}$$

we shall assume that for each p in $1 \le p \le \infty$ K is a compact linear operator in L_p, $f \in L_p$, and the corresponding homogeneous equation has no non-trivial solution in L_p. It follows then from the Fredholm theorem that a (unique) solution $y \in L_p$ exists for each $f \in L_p$.

The Galerkin method, in which an approximate solution y_h is sought in a finite-dimensional space $S_h \subset L_\infty$ (see Section 2 for details), is a well understood numerical method for the solution of (1.1). Here we are more concerned with the iterated variant of the Galerkin method, i.e. with the approximation $y_h^{(1)}$ obtained by substituting the Galerkin approximation y_h into the right-hand side of the integral equation, giving