SOME PROBLEMS OF SPECTRAL THEORY

Werner Ricker

As noted by several authors (eg. [7], [8]), the spectrality of operators with spectrum contained in $\mathbb{R}$ or the unit circle $\mathbb{T} = \{z \in \mathbb{C};\ |z| = 1\}$, can often be determined by an examination of the groups $\{e^{isT};\ s \in \mathbb{R}\}$ and $\{T^n;\ n \in \mathbb{Z}\}$, respectively. The problem is to determine when the Stone Theorem holds for these groups, that is, to determine when they are the Fourier-Stieltjes transform of a spectral measure defined on the dual group. For our purposes, it suffices to consider Stone's Theorem for the pair of (dual) groups $\mathbb{Z}$ and $\mathbb{T}$ (see [8], [11] for example).

Let X be a locally convex Hausdorff space, always assumed to be quasi-complete. The space of continuous linear operators on X with the strong operator topology is denoted by $L(X)$. The spectrum of an operator $T \in L(X)$ is taken in the sense of [11; p.270]. The σ-algebra of Borel sets of $\mathbb{T}$ is denoted by $\mathcal{B}$. Let $\mathcal{P}$ denote the space of trigonometric polynomials in $C(\mathbb{T})$ If $\psi \in C(\mathbb{T})$, then $\hat{\psi}$ denotes its Fourier transform.

<u>STONE'S THEOREM</u>. *Let the space X be barrelled and $T \in L(X)$ have an inverse in $L(X)$. Suppose that one of the following conditions is satisfied.*

(i) *For each $x \in X$, the subset*

$$A_T(x) = \{p(T)x;\ p \in \mathcal{P},\ \|p\|_\infty \le 1\},$$

of X, is relatively weakly compact.

(ii) *The group $\{T^n;\ n \in \mathbb{Z}\}$ is an equicontinuous part of $L(X)$ and*

$$B_T(x) = \left\{k^{-1}\sum_{m=0}^{k-1}\ \sum_{n=-m}^{m} \hat{\psi}(-n)T^n x;\ k = 1,2,\ldots,\ \psi \in C(\mathbb{T}),\ \|\psi\|_\infty \le 1\right\}$$