A LINEARIZED ELLIPTIC FREE BOUNDARY VALUE PROBLEM

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This is a report on joint work with John van der Hoek. We consider the flow of an irrotational inviscid and incompressible fluid under a thin body of convex plan form at a non-uniform small clearance from a plane ground surface. The problem is relevant to vehicle aero-dynamics, especially for racing cars. It was brought to our attention by E.O. Tuck who considered certain aspects of the problem in [3].

Following Tuck we take the body to be fixed and the flow to have a uniform velocity at infinity of U in the positive x-direction. The plan form of the body is assumed to be a bounded convex domain Ω in $\mathbb{R}^2$ which is symmetric with respect to the x-axis and has a smooth boundary $\partial\Omega$.

For each point $q \in \partial\Omega$ let $\theta = \theta(q)$ denote the angle measured in the anticlockwise direction between the positive x-axis and the outward unit normal $\nu = \nu(q)$ at q, with $-\pi \leq \theta(q) \leq \pi$. See the diagram.

The leading and trailing edges of Ω determined by the transition points $p = (a, b)$ and $\overline{p} = (a, -b)$ in $\partial\Omega$ are the sets

$$\Gamma_L(p) = \{q \in \partial\Omega : |\theta(q)| \geq |\theta(p)| , q \neq p \text{ or } \overline{p}\} \qquad \text{and}$$

$$\Gamma_T(p) = \{q \in \partial\Omega : |\theta(q)| \leq |\theta(p)| , q \neq p \text{ or } \overline{p}\} .$$

The distance between the body and the ground surface at the point $(x, y) \in \overline{\Omega}$ is $h(x, y)$. We assume that h is a positive smooth function