SQUARE ROOTS OF OPERATORS AND APPLICATIONS TO HYPERBOLIC P.D.E.'s

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INTRODUCTION

Throughout this paper H denotes a complex Hilbert space and V denotes a dense subspace, also with a Hilbert space structure, which is continuously embedded in H. The two norms are denoted $\|\cdot\|$ and $\|\cdot\|_V$.

For each $t \in [0,t_1]$, J_t denotes a sesquilinear form with domain $V \times V$ which satisfies

$$0 \leq J_t[u,u] \qquad , \text{ and}$$

$$\kappa\|u\|_V^2 \leq J_t[u,u] + \|u\|^2 \leq M\|u\|_V^2$$

for all $u \in V$, where κ and M are positive numbers, independent of t and u.

The associated operators T_t are the operators with largest domains satisfying

$$J_t[u,v] = (T_t u,v)$$

for all $v \in V$. They are non-negative self-adjoint operators and have non-negative square roots $T_t^{\frac{1}{2}}$ with domains $\mathcal{D}(T_t^{\frac{1}{2}}) = V$. Indeed

$$J_t[u,v] = (T_t^{\frac{1}{2}}u, T_t^{\frac{1}{2}}v)$$

for all u and v in V. See [7] for details.